SUMS OF PRODUCTS OF THE TERMS OF THE GENERALIZED LUCAS SEQUENCE $\{V_{kn}\}$

Emrah Kılıç*, Yücel Türker Ulutaş† and Neşe Ömür‡

Received 09 : 06 : 2010 : Accepted 11 : 10 : 2010

Abstract

In this study we consider the generalized Lucas sequence $\{V_n\}$ with indices in arithmetic progression. We also compute the sums of products of the terms of the Lucas sequence $\{V_{kn}\}$ for positive odd integers $k$.

Keywords: Second order linear recurrence, Fibonomial coefficients.


1. Introduction

The binary linear recurrence $W_n = W_n(a, b; p, q)$ is defined as follows for $n > 1$,

$$W_n = pW_{n-1} + qW_{n-2},$$

where $W_0 = a, W_1 = b$.

The Binet formula for $\{W_n\}$ is

$$W_n = A\alpha^n + B\beta^n,$$

where $A = \frac{b-a\beta}{\alpha-\beta}, B = \frac{a-\alpha b}{\alpha-\beta}$ and $\alpha, \beta = \left( p \pm \sqrt{p^2 + 4q} \right) / 2$.

For $n > 1$ and a fixed positive integer $k$, the terms of $\{W_{kn}\}$ satisfy the recursion [6, 7]:

$$W_{kn} = V_k W_{k(n-1)} - (-q)^k W_{k(n-2)},$$

*TOBB University of Economics and Technology, Mathematics Department, 06560 Ankara, Turkey. E-mail: ekilic@etu.edu.tr
†Kocaeli University, Mathematics Department, 41380 İzmit, Kocaeli, Turkey. E-mail: (Y. T. Ulutaş) turkery@kocaeli.edu.tr (N. Ömür) neseomur@kocaeli.edu.tr
‡Corresponding Author.